A Framework for Success in Implementing Mathematical Modelling in the Secondary Classroom

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A framework to support successful implementation of mathematical modelling in the secondary classroom was developed from transitions between stages in the modelling process and the cognitive activities associated with these. This framework is used to analyse implementation of a task with a Year 9 class. Cognitive activities engaged in during the task and competencies and technological knowledge required to complete the task successfully are identified. This framework can be used by teachers, researchers, and curriculum designers to design tasks and predict where in a given task blockages occur, hence allowing advance consideration of scaffolding for in-the-moment classroom decisions.

Internationally the field of applications and mathematical modelling in education features prominently in every continent and research into teaching and learning through applications and mathematical modelling is currently being pursued with renewed vigour in many parts of the world (Kaiser, Blomhoj, & Sriraman, 2006), boosted by the 14th International Commission on Mathematical Instruction (ICMI) study on applications and modelling in mathematics education held in 2004. The recently published volume by Blum, Galbraith, Henn, and Niss (2007) from this study contains an up-to-the-minute account of progress and challenges within the field. International initiatives currently addressing these challenges include, for example, the Organisation for Economic Co-operation and Development Programme for International Student Assessment (OECD PISA) project, which includes the following within its assessment domains.

> An individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage in mathematics in ways that meet the needs of that individual’s life as a constructive, concerned, and reflective citizen. (OECD, 2003, p. 15)

This implies engaging with mathematics across a variety of situations and contexts. In countries both within and outside the OECD such statements are associated with ongoing discussion about the design of curricula, and in particular the role of mathematical modelling, applications, and relations to the real world in the teaching and learning of mathematics (Blum et al., 2007). However, Turner (2007, p. 440) raises concerns about the extent to which the mathematical thinking that underpin such mathematical modelling tasks is really valued by those overseeing curriculum and instruction in various countries considering “the level of complexity of the mathematical modelling activities that 15-year-old students can cope with … seems to be rather low”. Turner also asks: “How can teachers be more effectively empowered to explore and promote the mathematical thinking underlying these tasks, and what kinds of teaching and learning activities will be most effective in facilitating this kind of mathematical thinking among 15-year-old students?”

Within Australasia modelling is advocated in curriculum documents from the primary years (e.g., Victorian Curriculum and Assessment Authority (VCAA), 2005) through to the

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upper end of secondary (e.g., Ministry of Education, 1992; Queensland Board of Senior Secondary School Studies (QBSSSS), 2000). Evaluations of curricular initiatives become confused when there are idiosyncratic interpretations, which muddy notions of authentic practice in the field. It is of continuing importance that initiatives claiming *mathematical modelling* as their focus, are presented in terms of frameworks, criteria, and alternatives that are endorsed by the international community of practice.

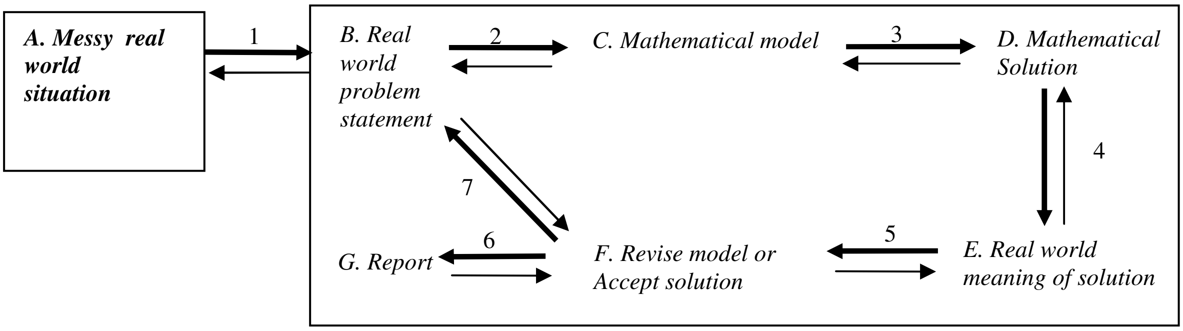
Given the various idiosyncrasies associated with some localised curricular initiatives (including Australian) we wish to be clear about meanings and interpretations ascribed to terms such as *applications* and *mathematical modelling* in our work. Our meanings are consistent with those adopted by the International Community for the Teaching of Mathematical Modelling and Applications (ICTMA), which is an Affiliated Study Group of the ICMI. Simply put, with *applications* we tend to focus on the direction (mathematics → reality). "Where can I use this particular piece of mathematical knowledge?" On the other hand with *mathematical modelling* we focus on the reverse direction (reality → mathematics). "Where can I find some mathematics to help me with this problem?"

The term mathematical modelling itself, as it is used in curricular discussions and implementations has different, although clearly delineated, interpretations. One interpretation sees mathematical modelling as motivating, developing, and illustrating the relevance of particular mathematical content (e.g., Chinnappan & Thomas, 2003). A second perspective views use of applications and modelling as an end in itself for educational purposes not a means for achieving some other mathematical learning end. The models and modelling perspectives of Lesh and English (2005), for example, although clearly associated with the first interpretation, extend beyond to include elements of the second. Our own approach sees the second interpretation as encompassing the first. Both approaches agree that modelling involves some total process that encompasses formulation, solution, interpretation, and evaluation as essential components.

The Modelling Process and Modelling Competencies

As interests in teaching and learning are central, our theoretical framework for studying modelling is oriented towards the problem solving individual to give not only a better understanding of what students do when solving (or failing to solve) modelling problems, but also a better basis for teachers' decision making and interventions. Figure 1, modified from Galbraith and Stillman (2006), encompasses both the task orientation of many diagrammatic representations of the modelling cycle and the need to capture what is going on in the minds of individuals as they work on modelling tasks. This latter focus has also led to a reduction in the number of stages identified specifically as other researchers (e.g., Borromeo Ferri, 2006) have pointed out that fewer are of more use in a schooling context.

The respective entries A-G represent stages in the modelling process, where the thicker arrows signify *transitions* between the stages, and the total solution process is described by following these arrows clockwise around the diagram from the top left. It culminates either in the report of a successful modelling outcome, or a further cycle of modelling if evaluation indicates that the solution is unsatisfactory in some way. The kinds of mental activity that individuals engage in as modellers attempt to make the transition from one modelling stage to the next are given by the broad descriptors of cognitive activity 1 to 7 in Figure 1. The light arrows that are in the reverse direction to the modelling cycle are included to emphasise that the modelling process is far from linear, or unidirectional, and to indicate the presence of reflective metacognitive activity (Maaß, 2006).



1. Understanding, structuring, simplifying, interpreting context
2. Assuming, formulating, mathematising
3. Working mathematically
4. Interpreting mathematical output
5. Comparing, critiquing, validating
6. Communicating, justifying (if model is deemed satisfactory)
7. Revisiting the modelling process (if model is deemed unsatisfactory).

Figure 1. Modelling Process.

It is imperative that we identify specifically activities with which modellers need to have competence in order to apply mathematics successfully particularly in settings where there is increasing access to electronic technologies. By "competency" is meant the capacity of an individual to make relevant decisions, and perform appropriate actions in situations where those decisions and actions are necessary to enable success.

> *Mathematical modelling competency* means the ability to identify relevant questions, variables, relations or assumptions in a given real world situation, to translate these into mathematics and to interpret and validate the solution of the resulting mathematical problem in relation to the given situation, as well as the ability to analyse or compare given models by investigating the assumptions being made, checking properties and scope of a given model (Niss, Blum, & Galbraith, 2007, p. 12).

We elaborate how these components of modelling competency are realised within the research settings that have provided data for this paper.

From Theory to Empiricism

The transitions arising from our theoretical framework (Figure 1) served as a structural framework for indentifying student blockages in transitions as students undertook various modelling and application tasks. Initially the contents of the respective transition sections were empty, except for the bold headings of Figure 2. Intensive data were generated from implementations of two teacher designed tasks at one school where modelling and the use of technology were an integral part of classroom practice in order to develop our first result, an "emergent framework" (Galbraith, Stillman, Brown, & Edwards, 2007), from empirical study. The resulting emergent framework was then refined and tested by examining the implementations of a different task and a revised version of one of the first tasks in a second school (Galbraith & Stillman, 2006). The task was revised by the researchers in collaboration with the teacher to suit the different motivation towards real world tasks and technology use and time frame of the teacher in a different school setting. The resulting refined transitions framework is shown in Figure 2. The empirics gave rise to case specific categories and generalisations of these from the various elements in each transition section. Our research indicates there is potential for blockages to occur when any of these component activities have to be undertaken.

1. MESSY REAL WORLD SITUATION → REAL WORLD PROBLEM STATEMENT:
1.1 Clarifying context of problem
1.2 Making simplifying assumptions
1.3 Identifying strategic entit(ies)
1.4 Specifying the correct elements of strategic entit(ies)
2. REAL WORLD PROBLEM STATEMENT → MATHEMATICAL MODEL:
2.1 Identifying dependent and independent variables for inclusion in algebraic model
2.2 Realising independent variable must be uniquely defined
2.3 Representing elements mathematically so formulae can be applied
2.4 Making relevant assumptions
2.5 Choosing technology/mathematical tables to enable calculation
2.6 Choosing technology to automate application of formulae to multiple cases
2.7 Choosing technology to produce graphical representation of model
2.8 Choosing to use technology to verify algebraic equation
2.9 Perceiving a graph can be used on function graphers but not data plotters to verify an algebraic equation
3. MATHEMATICAL MODEL → MATHEMATICAL SOLUTION:
3.1 Applying appropriate symbolic formulae
3.2 Applying algebraic simplification processes to formulae to produce more sophisticated functions
3.3 Using technology/mathematical tables to perform calculation
3.4 Using technology to automate extension of formulae application to multiple cases
3.5 Using technology to produce graphical representations
3.6 Using correctly the rules of notational syntax (whether mathematical or technological)
3.7 Verifying of algebraic model using technology
3.8 Obtaining additional results to enable interpretation of solutions
4. MATHEMATICAL SOLUTION → REAL WORLD MEANING OF SOLUTION:
4.1 Identifying mathematical results with their real world counterparts
4.2 Contextualising interim and final mathematical results in terms of RW situation (routine → complex versions)
4.3 Integrating arguments to justify interpretations
4.4 Relaxing of prior constraints to produce results needed to support a new interpretation
4.5 Realising the need to involve mathematics before addressing an interpretive question
5. REAL WORLD MEANING OF SOLUTION→ REVISE MODEL OR ACCEPT SOLUTION:
5.1 Reconciling unexpected interim results with real situation
5.2 Considering Real World implications of mathematical results
5.3 Reconciling mathematical and Real World aspects of the problem
5.4 Realising there is a limit to the relaxation of constraints that is acceptable for a valid solution
5.5 Considering real world adequacy of model output globally

Figure 2. Refined framework for identifying student blockages in transitions.

Practical Applications: Using the Framework

With respect to the questions raised by Turner (2007) about how we can promote the mathematical thinking underlying modelling tasks, our attention turns to the use of this transitions framework and Figure 1 to examine the implementation by a teacher and experience by students of a real world task. In order to identify the mathematical thinking that is being promoted by the task and the competencies required for a successful experience, we answer the following questions:

- What kinds of cognitive activites are students engaging in when the task is structured and implemented in this manner?
- With respect to the task as a modelling experience, what competencies (mathematical/modelling/technological) must students have to complete the task successfully?

The implementation of the task, *The Bungee Experience*, was chosen to illustrate the utility of the framework for three reasons. Firstly, it has fewer transition elements than more complex tasks. It is one of a series of modelling and applications tasks used in this year level at the school concerned so it is not necessary for all tasks to promote all elements of Figure 2 but it is important for pedagogical decisions to be based on informed judgements about the nature of the elements it does include or exclude. Secondly, the task, in various forms, has been used as both a teaching task and an assessment task as in this instance. The possibility arises that the teacher's purpose for the task also affects the elements that are promoted and the competencies required. Thirdly, this particular implementation was followed by a lesson reflecting on the model the students had used.

The Bungee Experience: Barbie has turned 40. Her friend Ken has given her an extreme sports experience, part of which is an afternoon's bungee jumping. Your task as the operator is to CALCULATE the length of Bungee Cord Barbie will need to jump from the given height, off the Bungee tower. Remember there is concrete below and we don't want to mess up Barbie's hair.
During the next two maths periods your team will:
1. Conduct measurements in the classroom to determine a model that links the fall distance to the number of rubber bands used for a shock cord.
2. Record your data, the graph for the data, and your linear equation.
3. Test your model by predicting the requirements for a fall from an unknown height.
(This height was announced later. Suggestions were provided as to how to collect data and display results. Students used a doll, usually Barbie or a toy such as Poombah the Warthog for the Bungee Jumper.)

Method

The task was implemented in a Year 9 class of 21 students during one 100-minute double period. The reflection lesson comprised the next 50 minute mathematics lesson two days later. Two video cameras were set up in the classroom. These were mainly focused on the class as a whole at the beginning of the double lesson and then on the collaborative activity of two focus pairs of students. At times, critical incidents involving other students were also videotaped. A third focus pair of students were audio-recorded. One camera was used to record the the reflection lesson and the jump phase of the implementation lessons when the class went outside and tested their bungee chords. Scripts from the 10 groups and rough working sheets from focus groups were collected. Five students participated in post-task interviews. Only one of these students was from the focus groups. Field notes were also made by the researchers during and immediately after the lessons.

All audio and video data were transcribed for analysis. The transcripts, in conjunction with the video recordings, were analysed at a macrolevel to identify episodes where students encountered and resolved (or otherwise) blockages between the identified transitions. Each episode was coded using elements of the transitions framework in Figure 2 then subjected to intense microanalysis to see if it shared the same characteristics as the elements of the framework identified previously or if further elements needed to be added. There were none. At the end of this process a framework showing the potential blockages was produced. Finally, typical instances of the cognitive actvities engaged in by students in the task and the competencies that underpin successful transitions from one modelling phase to the next were identified.

Results

Figure 3 shows the elements in each transition that were identified in this implementation of the task. Each element has two parts where key (generic) categories in the transitions between phases of the modelling cycle are indicated (in regular type), and illustrated (in capitals) with reference to the task. Cognitive activities associated with the elements of transitions identified in Figure 3 are: understanding, simplifying, interpreting context; assuming, formulating, mathematising; working mathematically; interpreting mathematical output; and comparing, critiquing, validating. Evidence for selected examples of these activities is presented in the analysis of transitions that follows. Finally, the competencies for carrying these out successfully are identified.

1. MESSY REAL WORLD SITUATION → REAL WORLD PROBLEM STATEMENT:
1.1 Clarifying context of problem [WATCHING DEMONSTRATION & DISCUSSING PROBLEM SITUATION]
1.2 Making simplifying assumptions [ELASTIC LIMIT NOT EXCEEDED; AERODYNAMICS OF TOYS CAN BE IGNORED]
2. REAL WORLD PROBLEM STATEMENT → MATHEMATICAL MODEL:
2.1 Identifying dependent and independent variables for inclusion in algebraic model [*FALL DISTANCE* AND *NUMBER OF ELASTIC BANDS* – WHAT CONTROLS WHAT]
2.3 Representing elements mathematically so formulae can be applied [POINTS]
2.4 Making relevant assumptions [LINEAR MODEL APPROPRIATE EVEN WHEN DATA POINTS APPEAR TO FOLLOW CURVE]
2.5 Choosing technology to enable calculation [RECOGNISING HAND METHODS ARE NOT SUFFICIENT]
2.7 Choosing technology to produce graphical representation of model [GRAPHING CALCULATOR WILL GENERATE PLOT OF *FALL* DISTANCE FOR DIFFERENT *NUMBERS OF RUBBER* BANDS]
3. MATHEMATICAL MODEL → MATHEMATICAL SOLUTION:
3.1 Applying appropriate formulae [EG. LINEAR MODEL TO FIND PREDICTED NUMBER OF BANDS]
3.3 Using technology/mathematical tables to perform calculation [SUCCESSFUL CALCULATION OF GRADIENT]
3.5 Using technology to produce graphical representations [EFFECTIVE USE OF GRAPHING CALCULATOR STATPLOT]
3.8 Obtaining additional results to enable interpretation [PLOTTING EXTRA VALUES TO TEST HUNCHES]
4. MATHEMATICAL SOLUTION → REAL WORLD MEANING OF SOLUTION:
4.1 Identifying mathematical results with their real world counterparts [INTERPRETING PREDICTION VALUE]
4.2 Contextualising interim and final mathematical results in terms of RW situation (routine versions) [GRADIENT MEASURES HOW FAR IT WILL FALL PER BAND]
4.3 Integrating arguments to justify interpretations [PRESENTING REASONED CHOICE FOR METHOD OF FINDING EQUATION OF LINEAR MODEL]
4.4 Relaxing of prior constraints to produce results needed to support a new interpretation [CAN USE POINTS INVOLVING HALF BANDS TO FIND GRADIENT OF LINEAR MODEL]
5. REAL WORLD MEANING OF SOLUTION → REVISE MODEL OR ACCEPT SOLUTION:
5.1 Reconciling unexpected interim results with real situation [RECONCILING THE RESULTS OF TESTING THEIR PREDICTIONS 26 BANDS WITH BARBIE VERSUS 26 WITH POOMBAH]
5.2 Considering Real World implications of mathematical results [LOCAL – DO INDIVIDUAL CALCULATIONS/GRAPHS ETC MAKE SENSE WHEN TRANSLATED TO REAL WORLD MEANINGS?]
5.3 Reconciling mathematical and Real World aspects of the problem [SIGNIFICANCE OF Y-INTERCPET IN LINEAR MODEL & HOW IT COULD BE USED TO PARTIALLY EVALUATE MATHEMATICAL EQUATION CONSTRUCTED]
5.5 Considering real world adequacy of model output globally [MODEL PROVIDES ALL ANSWERS TO RW PROBLEM & EXTENDS TO OTHER SITUATIONS]

Figure 3. Framework showing transition elements in Barbie Bungee implementation.

Transitions

Messy real world situation → Real world problem statement. In this implementation this transition presented no blockages to students' progress. The teacher demonstrated how to topple the doll and the attaching of bands to the doll's ankles (1.1). In other implementations viewed by the researchers, difficulties arose when students attempted data collection with the doll upside down hanging from her toes or they threw the doll rather than toppled her from a standing position. A major assumption (1.2) is that the bands will stretch at a constant rate and not exceed their elastic limit when the model is used to extrapolate well beyond the set of collected data (maximum drops of around 2m in a classroom). This assumption was debated by students during the reflection lesson.

Ray: But eventually shouldn't the rubber bands snap therefore it can't [interrupted]
Teacher: It would if the weight component enables it, and ultimately the weight of the rubber band itself causes problems. There are some problems in the linear model …
Dale: Instead of snapping it could get 0.3 cm longer by bending so our calculations

Ray: [interrupting] But eventually it is going to bend. It is going to snap.
Dale: Stretch!
Ray: It is not a linear model!
Tine: Rubber bands stretch.
Dale: That would make our distances vary.
Teacher: It has to stretch. It gets to what is called its elastic limit and then the linear part changes, okay? … but in terms of 8 bands we would probably get away with a good approximation.

In this implementation it was also assumed that the aerodynamic characteristics of toys such as Poombah would have negligible impact (1.2). When Poombah dropped about half the distance of a Barbie doll with the same length Bungee cord, one student suggested that perhaps it was a weight difference. No students raised its aerodynamic characteristics. From a modelling perspective, some of the responsibility for elements of formulation such as identifying the strategic entity and specifying its elements were removed from the students as they were told they were to collect fall distance data.

Real world problem statement → Mathematical model. Even though students were told to collect fall distances for 3 to 8 rubber bands, not all students easily recognised which of these was the dependent and which the independent variable in the situation (2.1). Bea and Sue for example, had them reversed initially and remained unsure they were correct when they swapped them. Students choice of points for calculating gradients for lines of best fit (2.3) caused some delays in moving on from their models when algebraic manipulation produced equations with intercepts that clearly were too large. Although the task setter had already made many decisions in this transition for students by specifying a linear model be made and choosing the technology to use for a plot (graph paper), some students (e.g., Evan) decided to question whether they should use a linear model as their plot showed their data were curved (2.4) or chose to use a graphing calculator to check their hand drawn plot (2.7). It did not occur to Evan to check his data rather than merely question the model.

Mathematical model → Mathematical solution. Sue and Bea did not use an appropriate linear model when they calculated the number of bands for their prediction (3.1) believing that they should choose plotted points in such a manner that the line would pass through the origin. They later told the teacher this was because they expected a y-intercept of zero as Barbie would stand and not fall at all if the length of the Bungee cord was zero. Unlike previous implementations of the task, no students obtained additional results or attempted in some way to test their models before the Jump Phase, although Sue suggested they test their model using 9 bands but they failed to do so (3.8).

Mathematical solution → Real world meaning of solution. Possible dilemmas for students in this transition occur when students do not identify mathematical results such as the gradient and the y-intercept with their real world counterparts (4.1) and when they need to contextualise interim and final mathematical results in terms of the real world situation (4.2) for example, when predicting a shock cord length for their test jump outside the classroom. The doll was to be dropped so as to stop as close to the ground as possible. When the students finally found their mathematical result for the predicted number of bands, decisions had to be made about whether they should round up, truncate their answer, or over or under estimate. The real world implication (5.2), that rounding up or over estimating would mean the doll would hit the ground was foreseen by four groups.

Tony: So we need 24 rubber bands.
Reg: Yeah. Should we go with 23 just to be safe? Or should we just go 24.

Tony: No 24 because look point [Calculator shows 24.3950762]
Reg: All right. So we rounded down because if it hits the ground we have to clean the tennis courts.

Two other groups rounded their result up, in keeping with expected classroom practice, clearly not considering the implications in the context, with Tine, Lil, and Ally showing their prediction resulted in a fall distance of 441.3 cm exceeding the jump distance of 440 cm. Others such as Ray fortuitously rounded down giving no thought to the context.

Researcher: 27.16 recurring. So why did you round down? Do you just always do that?
Ray: I don't know I just rounded down. Should I have rounded up? Well, even if it was a 9 it would have made it 27.2 and that is still not enough to round up anyway.

Real world meaning of solution→ Revise model or accept solution. As students tested their predictions many were puzzled as to why these were wrong. They had difficulty reconciling the jump results with the mathematics of the situation (5.1), which they did not use to evaluate their models. Ray and Joe's Barbie made an almost perfect jump with 27 bands whereas Di and Ash's Barbie with 28 bands was about 70 cm short. There was no discussion of the difference in their models, $y = 15x + 32.5$ and *distance* (cm) = 14 × *rubber bands* + 38.3, although the teacher brought to the students' attention the role of the gradient in determining how many more bands to add after he allowed them a third jump. In the reflection lesson student discussion teased out the meaning of the gradient.

Teacher: What does the gradient measure? In Barbie's case what does the gradient measure?
Dale: Rise divided by run.
Teacher: Yeah, that's how you calculate it. What does it actually physically mean?
Tony: How many rubber bands.
Ray: How far it would fall per rubber band.

No students showed evidence of realising the significance of the y-intercept in their mathematical model and how it could be used to evaluate partially the mathematical equation they had constructed (5.3). This was discussed in the following lesson when the students reflected on the model they had made. In the exchange below they are discussing a linear equation of the form $y = \mathrm{m}x + \mathrm{c}$ where c is 25.4.

Teacher: What was the 25.4 in Barbie or, if you had Poombah (the Warthog) it was less.
Tine: The length of Barbie.
Teacher: It was the length of Barbie. Now I had this discussion with a couple of people [Bea snd Sue] about how many rubber bands, how far Poombah or it would fall. Some said, "Zero", but the thing is that Barbie would fall … So she would still be hanging by her toenails… a lot of the times you get c values that don't make a lot of sense. In Barbie's case it did.

The reflection lesson continued with students suggesting and discussing many other applications of linear models (5.5) such as mobile phone plan charges, cost of purchasing concert tickets over the phone, council service charges, and electricity costs.

Competencies

Modelling and mathematical competencies identified as being required for the task, and graphing calculator technological knowledge required for the task are presented in Figure 4.

Modelling and mathematical competencies:

To identify from the available information what is relevant and what is irrelevant,

To make simplifying assumptions about the situation,

To recognise relevant variables and to mathematise these,

To make relevant assumptions to enable mathematics to be applied,
To select technology where needed to enable or check calculations,
To chose appropriate methods of representing, checking and testing the model,
To select and apply appropriate formulae (e.g., general form of linear model: $y = ax + b$., gradient: $\Delta y \div \Delta x$),
To use technology appropriately to perform calculations,
To use mathematical knowledge to solve the problem,
To obtain additional results to enable interpretation,
To link mathematical results with their corresponding real world components,
To generalise or extend solution,
To critically check results with the real situation,
To consider implications of decisions and results.

Technological knowledge needed for effective use of a graphing calculator:

To know data can be entered into LISTS and LIST data can be plotted,
To use of Homescreen of a calculator to perform calculations,
To know how to Plot correctly and effectively.

Figure 4. Modelling and mathematical competencies and technological knowledge for *Barbie Experience*.

Practical Implications for the Curriculum and Classroom

The framework we have devised allows a researcher, teacher, or curriculum designer to identify the particular competencies that students would need in order to complete a particular modelling task successfully. By identifying difficulties with generic properties the possibility arises of teachers/researchers/curriculum designers being able to predict where in given problems, blockages of different types might be expected. This understanding then contributes to the planning of teaching, in particular the identification of necessary prerequisite knowledge and skills (including modelling competencies), preparation of interventions for introduction at key points if required, and the scaffolding of significant learning episodes. As well as identified blockages showing teachers what they may need to address to help students overcome blockages the framework also informs the teacher who is trying to move from dependent to independent modelling by students (Leiβ, 2005).

Although it is acknowledged in many curricula documents (e.g., Ministry of Education, 1992; OECD, 2003; QBSSSS, 2000; VCAA, 2005) that mathematical modelling is an essential component of secondary schooling, implementing this is no simple task. Considering how mathematics can be used to solve real problems, requires students to make decisions about many aspects of the task. Whilst this is an important part of the learning process, it can place the teacher in the position of needing to provide appropriate scaffolding "on the spot" when some unforseen blockage is encountered by one or more students. This can be a challenge for the most experienced teacher. Thus, both practising and pre-service teachers could benefit from the use of a tool.

By mapping the task and its intended implementation to the transitions Framework (Figure 3), prior to the actual implementation, teachers can identify the specific activities with which the student modellers need to have competence in order to apply their mathematical and technological knowledge successfully to the problem. Identifying potential blockages can inform planning of teaching. This does not mean making decisions for students to avoid their confronting blockages, rather it allows the teacher to be well prepared, expecting particular blockages and better supporting students to overcome these. It also gives teachers information on which to base decisions about the preparedness of their students to complete a particular task.

One task can be easily modified to suit a range of purposes using the Framework. These purposes vary and include: the intent of the mathematical modelling (e.g., as a vehicle to teach modelling competencies or to legitimate mathematical content), the time that can be allocated to the task, the purpose being assessment or learning focused, previous experience of the teacher in implementing modelling tasks, previous experiences of students with modelling tasks, the technological expertise of the students, and the mathematical knowledge of the students. By using the Framework, teachers and others can modify a task to suit their particular purpose and constraints.

Teachers and curricula designers wishing to implement a series of modelling tasks over the course of a year can use the Framework to ensure that, although not all elements will be addressed in every task, every element is included in as many tasks as necessary to develop students' modelling competencies. The incorporation of formulation and reflection activities are critical to developing modelling skills.

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